ME104 Lab 5: Motor Characterization

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1 Objective and Procedure

The objective of this lab was to study the characteristics of a permanent magnet (PM) brushed DC motor. We used an external hall effect sensor and an internal tachometer, monitored via an oscilloscope, to study some of the properties related to the motor's rotational speed. Additionally, we used a Digital Multi-Meter (DMM) to evaluate the current draw and the voltage across the motor at different operating points. Lastly, the terminal inductance and resistance of the motor were measured to provide all of the information necessary for characterizing the operation of the motor up to its nominal voltage.

2 Results

The following sections provide a detailed overview of the results that were collected for estimating various motor properties.

2.1 EMF as a Function of Motor Speed

A PMDC motor functions because current-carrying conductors in a magnetic-field experience a force proportional to their current and the field-strength. Similarly, a conductor moving in a magnetic field develops a voltage potential - this is known as EMF (electromotive force) or back-EMF in DC motors. The back-EMF indicates the fraction of input energy that results in useful work, so it is an important figure in characterizing motor performance.

To monitor the EMF as a function of motor rotational speed, the test-subject motor was coupled to a driving motor via a shaft-coupler. The input voltage to the driving motor was swept between 4-24V (for 10 data points) and the voltage across the driven motor was monitored with a DMM. The rotational speed was monitored via a hall-effect sensor, with the sensor output recorded on an oscilloscope with an oscilloscope probe. The recorded voltages are shown in table 1, and the response of the hall-effect sensor is shown in 1.

The data collected in this section contains the necessary information for several motor properties: the electric (and subsequently torque) constant relates the rotational speed to

Driving Input Voltage (V)	Driven EMF (V)
4	0.932 ± 0.007
6	2.575 ± 0.01
8	4.65 ± 0.04
10	6.50 ± 0.05
12	8.35 ± 0.06
14	10.31 ± 0.07
16	12.20 ± 0.08
18	14.19 ± 0.09
20	15.82 ± 0.1
22	17.79 ± 0.1
24	19.96 ± 0.1

Table 1: Back-EMF measured at the terminals of the driven motor at different input voltages to the driving motor.

EMF, and can be distilled from this information. The torque constant, in turn, yields the stall torque when paired with stall current. Lastly, the speed constant relates input voltage to motor speed, which is once again reflected in this data for 10 points.

2.2 Inductance

Inductance was measured by recording the motor response when driven with a square wave. We know from basic RLC circuit analysis, the natural frequency of an RLC circuit follows equation 1. The response of the system was recorded by an oscilloscope and processed in our analysis to evaluate the inductance of the motor. This is displayed in figure 2.

$$\omega^2 = \frac{1}{LC} \tag{1}$$

2.3 Stall Current

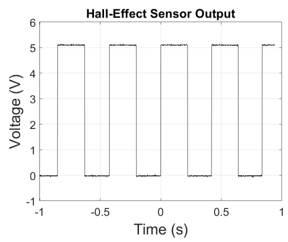
We measured the stall current by manually loading the motor shaft until stalled ($I_{stall} =$ 678 mA). This procedure was done rapidly to prevent overheating the motor, and the motor was set to cool down after the test.

2.4 No-Load Speed and Current

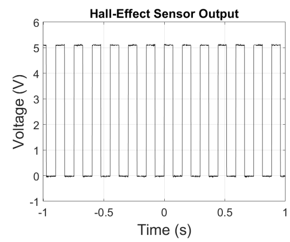
Similar to above, the no-load speed current was found by running the motor with the shaft un-impeded and measuring the current ($I_{no-load} = 64.8$ mA). We monitored the no-load speed using the hall-effect sensor, with the resulting plot shown in figure 3.

2.5 Mechanical Time Constant

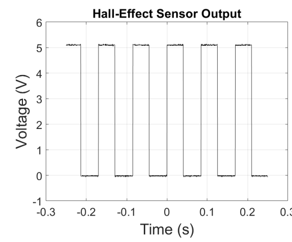
The mechanical time constant of the motor is the time needed for the motor to reach 63% of its steady-state speed. This measure is indicative of the response time of the motor, and was measured by monitoring the start-up of the motor through the AC Tachometer using the oscilloscope. The response as measured by the oscilloscope is provided in figure 4.



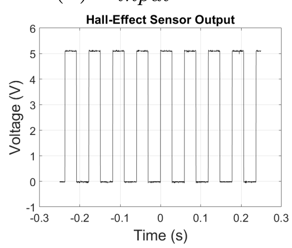
(a) $V_{input} = 4$ V



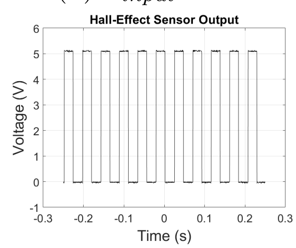
(b) $V_{input} = 6$ V



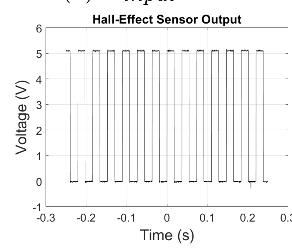
(c) $V_{input} = 8$ V



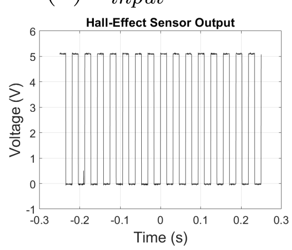
(d) $V_{input} = 10$ V



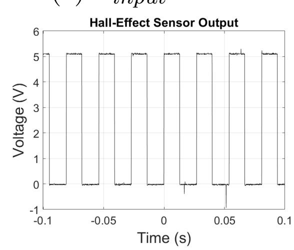
(e) $V_{input} = 12$ V



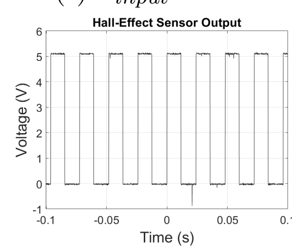
(f) $V_{input} = 14$ V



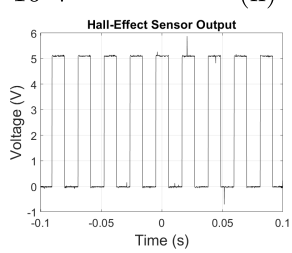
(g) $V_{input} = 16$ V



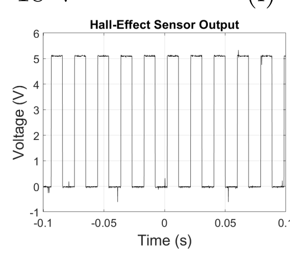
(h) $V_{input} = 18$ V



(i) $V_{input} = 20$ V



(j) $V_{input} = 22$ V



(k) $V_{input} = 24$ V

Figure 1: Data collected from hall-effect sensor at different input voltages. The frequency of the square signal is proportional to motor speed (rpm).

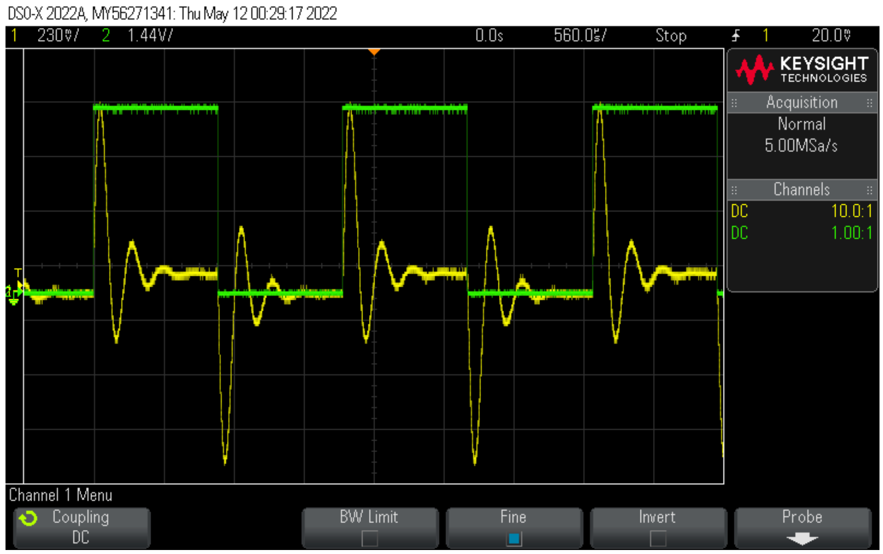


Figure 2: Oscilloscope data for motor response to square wave input

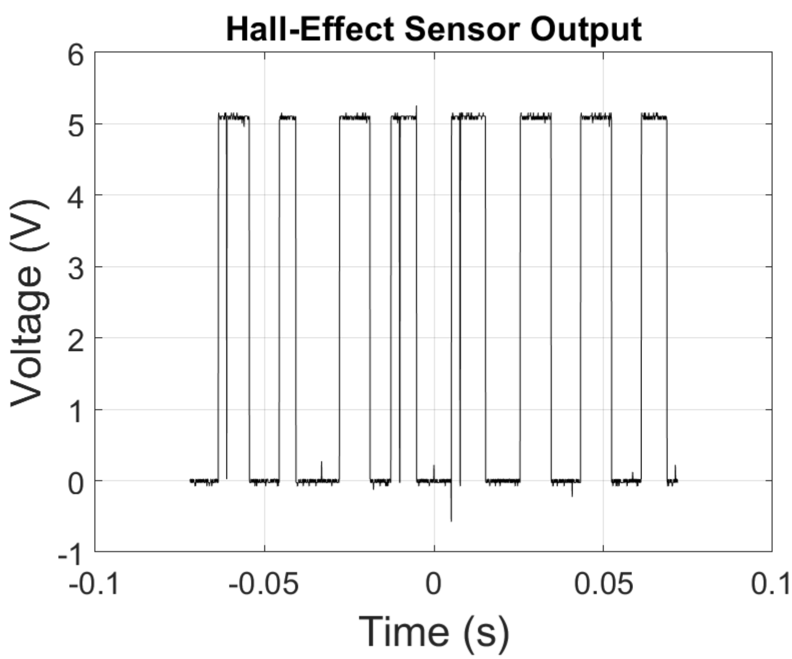


Figure 3: Data collected from hall-effect sensor at the no load operating point. The frequency of the square signal is proportional to motor speed (rpm).

Figure 4: AC tachometer data read by oscilloscope to find start-up time of motor

3 Analysis

3.1 Terminal Resistance

The resistance of the motor was determined to be $R = 49.3 \pm 0.8\Omega$. The resistance across the motor changed as the motor shaft was turned. This range of resistances varied from 0 to 150 Ω and 49.3 Ω was a value that was consistently measured. The error in this measurement was calculated from the DMM accuracy datasheet.

3.2 Terminal Inductance

In order to characterize the inductance of the motor, the response of the motor to a square wave input was studied. Analyzing the voltage across the motor measured by an oscilloscope (figure 2), we were able to determine the natural frequency of the circuit. We chose one of the step responses to analyze, as shown in figure 5.

To determine the natural frequency of the circuit, the damped frequency was determined from the period between peaks, which was determined from the signal. This was converted to the damped angular frequency using equation 2. The logarithmic decrement was calculated using equation 3, and from this, the damping ratio of the system was found with equation 4. From these, the natural frequency was determined from equation 5.

$$T_d = \frac{1}{2\pi\omega_d} \tag{2}$$

$$\delta = \ln\left(\frac{V_n}{V_{n+1}}\right) \tag{3}$$

$$\zeta = \frac{1}{\sqrt{1 + \left(\frac{2\pi}{\delta}\right)^2}} \tag{4}$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2} \tag{5}$$

Given a simple RLC circuit, the natural frequency is related to the inductance and capacitance of the circuit by equation 6. This was used with the given capacitance of $97 \pm 3nF$ to find inductance $L = 16 \pm 2mH$.

$$\omega^2 = \frac{1}{LC} \tag{6}$$

3.2.1 Error Propagation

To calculate error propagation in our inductance value, error was propagated from the oscilloscope voltage readings, through the logarithmic decrement, our damping ratio, and through the oscilloscope timing accuracy, to the natural frequency of the circuit, and finally

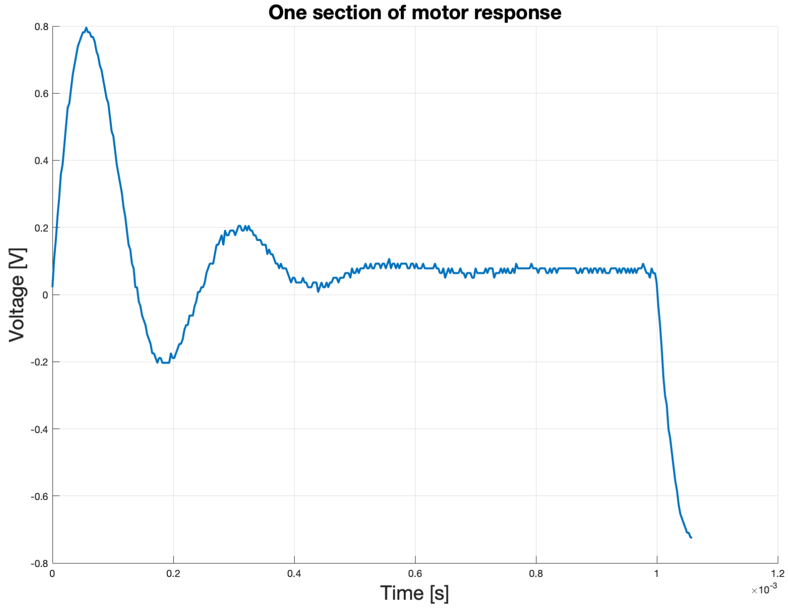


Figure 5: Section of motor step response analyzed for inductance

to the inductance. A sample calculation is provided later for reference. Note: While the DC bias in the step response was removed to calculate the logarithmic decrement and natural frequency of the system, it was retained in the error propagation to properly scale the error in the oscilloscope voltage reading.

3.3 EMF as a Function of Motor Speed

In order to plot EMF as a function of motor speed, the motor speeds must be extracted from the frequency response of the hall-effect sensor. To achieve this, a primitive edge-detecting MATLAB script was developed that identified the period of the square waves. More specifically, this script cycles through all data points and detects subsequent high-to-low or low-to-high transitions to estimate waveform frequency. To remain unaffected by sudden fluctuations, the edge-detecting algorithm takes multiple points into consideration when evaluating the square wave. The motor speed, in rpm, was then extracted from the following relation:

$$\begin{aligned} f &= \frac{1}{T} \\ v &= 60 \cdot f \text{ [rpm]} \end{aligned} \tag{7}$$

The error in determining the period was found from the time-error of the oscilloscope:

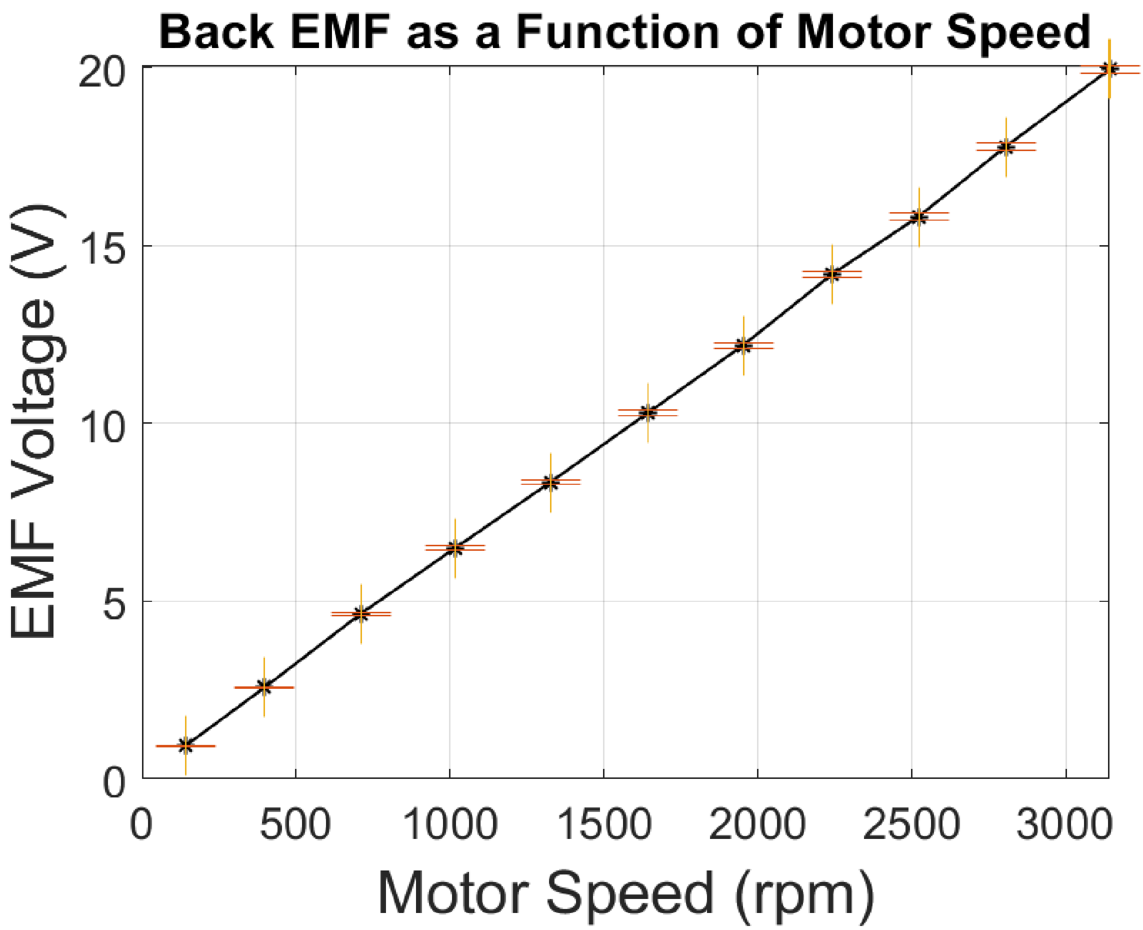


Figure 6: Back-EMF measured against the motor speed at different operating points. The resulting relationship is nearly linear up to the nominal motor voltage.

$\pm 30 ppm \cdot$reading, where the total error in period is equal to $60 \cdot \pm 30 ppm \cdot$reading 1 + reading 2 to account for the multiplication by 60s. Throughout our analysis, the accuracy of the DMM readings were found from the digital multi-meter data sheet and took the form

$$\text{Uncertainty} = C_1 \cdot \text{Reading} + C_2 \cdot \text{Resolution} \tag{8}$$

where C_1 and C_2 are empirical constants supplied by the manufacturer. The MATLAB script employed to evaluate these errors at each data point is featured in the appendix to this report. The resulting EMF vs Motor Speed plot is shown in figure 6.

3.4 Electric, Torque, and Speed Constants

The electric constant and torque constants are defined as

$$k_e = \frac{V_{EMF}}{\omega}\left[\frac{\text{mVs}}{\text{rad}}\right] \quad (9)$$
$$k_t = \frac{\tau}{I_{in}}\left[\frac{\text{Ncm}}{\text{A}}\right].$$

Importantly, these quantities are numerically equal when the units of $\frac{\text{Vs}}{\text{rad}}$ and $\frac{\text{Nm}}{\text{A}}$ are used. We observe that the electric constant is the slope of figure 6 - we therefore estimate this constant by fitting a line to the data points. Next, ensuring the appropriate use of units, we infer the torque constant from this value. Note that this process requires conversion of speed from units of rpm to rad/s, as handled in the MATLAB script. Finally, the error in the linear fit is found using the 95% intervals of confidence provided by the MATLAB fit library, and it is noted that relative errors are maintained in multiplication by a constant value. Using this procedure, we obtain the following values for the electric and torque constants:

$$k_e = 60.4 \pm 0.3\frac{\text{mVs}}{\text{rad}} \quad (10)$$
$$k_t = 6.04 \pm 0.03\frac{\text{Ncm}}{\text{A}}.$$

A third constant of interest is the speed constant, which is the ratio of motor speed to input voltage. The speed constant is equal to the reciprocal of the torque constant when expressed in appropriate units. Given that error does not propagate between reciprocals, the speed constant was found to be equal to $k_s = 158 \pm 0.003\frac{\text{rpm}}{\text{V}}$.

3.5 Electrical Time Constant

The electrical time constant is a time constant that governs the time required for the current draw into the motor to reach 63% of its final value. This was calculated using equation 11. With the previously determined values for resistance and inductance, the electrical time constant was determined to be $\tau_e = 0.32 \pm 0.05ms$. The error in this value was determined from the DMM uncertainty in the resistance and the error in oscilloscope readings that propagated into the inductance value.

$$\tau_e = \frac{L}{R} \quad (11)$$

3.6 Mechanical Time Constant and Rotor Inertia

The mechanical time constant of the motor governs how quickly the motor starts up and the time it takes to reach its full speed. Using the data from figure 4, the peaks were found using MATLAB to determine the frequency of the AC tachometer output. The change in frequency of the tachometer over time is superimposed on the voltage output of the tachometer in figure 7.

The mechanical time constant is defined as the time it takes for the speed of the motor to reach 63% of its maximum value, which was determined to be $\tau_m = 12.11 \pm 0.0004ms$.

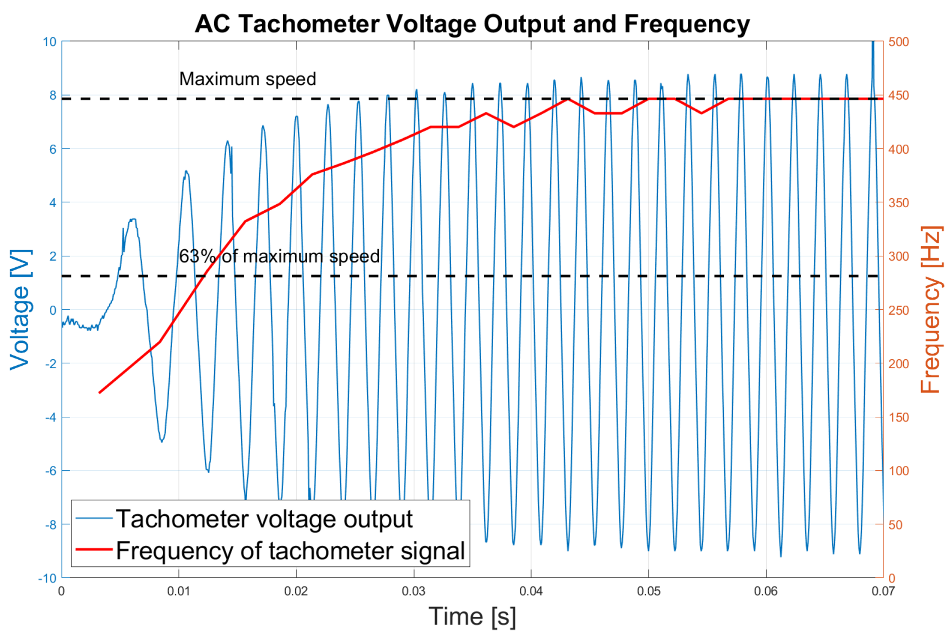


Figure 7: AC Tachometer voltage and frequency output for motor startup

On a related note, the rotor inertia is representative of the inertia of a motor. It describes the inertia of the rotor and is related to how the motor will accelerate when given an input signal. It can be found from the mechanical time constant, τ_m, resistance, R, and torque constant, k_t, with equation 12. For our motor, this was calculated to be $9.0 \pm 0.2g \cdot cm^2$

$$\tau_m = \frac{JR}{k_t^2} \tag{12}$$

3.6.1 Error Propagation

To find the uncertainty in our mechanical time constant, the uncertainty in the oscilloscope timing was calculated at the mechanical time constant. From the oscilloscope datasheet, the error in the timings is 30 ppm $\cdot$ (reading), which yields an uncertainty of

$\pm 0.0004ms$. This value is on a significantly lower order of magnitude than the mechanical time constant itself, so for other calculations, this error is neglected.

To find the error in the rotor inertia, error was propagated from the error in the torque constant and resistance. Error in the mechanical time constant was many orders of magnitude smaller and was negligible.

3.7 Stall Current and Torque

The stall current was measured to be $I_{stall} = 678 \pm 20\text{mV}$, with the uncertainty being evaluated using equation (8). Because the torque constant was estimated in section 3.4, the stall torque was found from the relation

$$\tau_{stall} = k_t \cdot I_{stall} = 4.09 \pm 0.1\text{Ncm}. \tag{13}$$

Here, the propagation of errors in multiplication was accounted for using the relation

$$\text{Relative Error in Multiplication } (z = x * y) = \frac{\delta z}{z} = \frac{\delta x}{x} + \frac{\delta y}{y}. \tag{14}$$

where the error in the torque constant was found in section 3.4.

3.8 No-Load Speed and Current

Similar to section 3.3, the speed was found from the frequency of the square waves using the edge-detection MATLAB script. With the time-uncertainty of the oscilloscope, this resulted in an estimated no load speed of $3350 \pm 0.0002\text{rpm}$. The no-load current was measured with the DMM and was equal to $I_{stall} = 64.8 \pm 1\text{mA}$. We note that the constants in equation (8) where updated according to the range of measured current as detailed in the manufacturer's data-sheet.

3.9 Torque vs Speed Curve

The theoretical torque vs speed curve for a PMDC motor is known to be a line connecting the no-load and stall operating points. With both of these points found in the above sections, we were able to plot the motor's torque vs speed curve as shown in figure 8. Note that the error bars reflect the uncertainties in the data points.

3.9.1 Maximum Power Consumption

For a curve of the form of figure 8, the maximum mechanical output power ($p = \tau \cdot \omega$) occurs when the speed is half of the no-load speed.

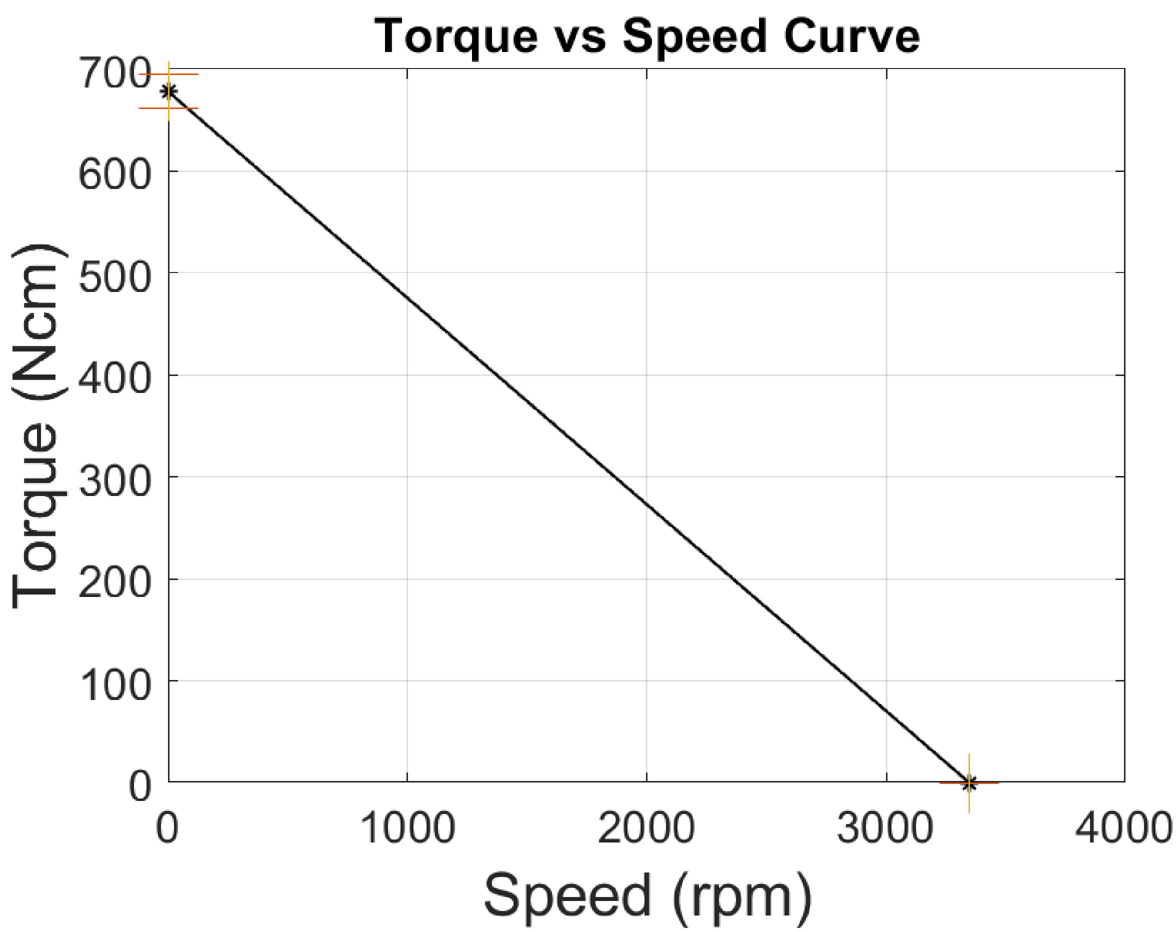


Figure 8: Motor torque plotted against motor speed.

$$\begin{aligned} P_{max} &= \frac{1}{2} \cdot \text{No-Load Speed} \cdot \frac{1}{2} \cdot \text{Stall Torque} \\ &= 3.59 \pm 0.1\text{W} \end{aligned} \tag{15}$$

where error propagation is handled via equation (14).

3.10 Maximum Efficiency

The maximum efficiency for a PMDC motor is approximated by the follow relationship in terms of the stall current and the no-load current - error propagation is handled similar to the above products, but we note that the error propagation cancels out between the squaring and square root operations, leaving only the quotient to be accounted for.

$$\begin{aligned} \eta_{max} &= \left(1 - \sqrt{\frac{I_o}{I_s}}\right)^2 \\ &= 0.477 \pm 0.004 \end{aligned} \tag{16}$$

3.11 Motor Datasheet

Table 2 shows all the motor characteristics compiled into a table.

4 Discussion

4.1 Terminal Inductance

The inductance of our motor was characterized by studying the transient response of the motor to a square wave, which served as a sequence of step inputs. While only one section of the response was required to determine the inductance of the motor, we realized that the choice of which section to analyze had an impact on our error calculations. Since conventional error propagation techniques normalize the error of any given measurement by the measurement itself, this can mean that a smaller measurement for any given error is more problematic. For example, a robot having a positional error of 2 centimeters after traveling a 1 meter path is much less problematic than a 2 centimeter error after a 5 centimeter path. However, in this case, this principle meant that our logarithmic decrement and damping ratio would have different uncertainties for the 5 volt and the 0 volt sections of the step (figure 5) despite both methods resulting in the same value for the decrement or damping. This phenomenon occurs because the response to the 5 volt step has a slight DC bias that settles at a higher voltage magnitude than the 0 volt step). For this experiment, we found that when analyzing the voltage of the 0 volt input, the error of the logarithmic decrement

Property	Symbol	Units	Value
Nominal Voltage	V_{nom}	V	24
Power Rating	P	W	5.0
Max. Power	P_{max}	W	3.6 ± 0.1
No-Load Speed	w_o	rpm	3350 ± 0.0002
No-Load Current	I_o	mA	64.8 ± 1
Stall Current	I_s	mA	680 ± 20
Stall Torque	τ_s	Ncm	4.09 ± 0.1
Max Efficiency	η_{max}	%	0.477 ± 0.004
Terminal Resistance	R	Ω	49.3 ± 0.2
Terminal Inductance	L	mH	16 ± 2
Electrical Time Constant	τ_e	ms	0.32 ± 0.05
Torque Constant	k_t	$\frac{Ncm}{A}$	6.04 ± 0.03
Electrical Constant	k_e	$\frac{mVs}{rad}$	60.4 ± 0.3
Speed Constant	k_s	$\frac{rpm}{V}$	158 ± 0.003
Mechanical Time Constant	τ_m	ms	12.11 ± 0.0004
Rotor Inertia	J	gcm^2	9.0 ± 0.2

Table 2: Empirically determined motor properties for a permanent magnet DC motor.

was greater than the value itself, which is not reasonable. To avoid this issue, we chose to analyze the response to the 5 volt step instead.

The discrepancy caused by the variability of the different portions of the response raises the question of whether or not our error propagation techniques are adequate. Ideally, our error propagation techniques should be impartial to the magnitude of the step input magnitude so as to avoid scenarios where error can seemingly decrease without any substantive changes in the analysis. In this case, a potentially more suitable choice of error normalization could be to normalize the error with respect to the difference of the voltage with the steady state response. However, this approach would yield the same issue where the error is on a greater order of magnitude than the inductance itself. In practice, the optimal solution would be to use an instrument with greater analog input accuracy for voltage readings instead.

4.2 EMF as a Function of Motor Speed and Motor Constants

The recorded EMF response as a function of shaft speed is very linear as seen in figure 6. This behavior closely matches the theoretical model of the motor consisting of an inductor, a resistor, and a back-EMF source, and allows for accurate prediction of EMF using the electric constant, $V_{EMF} = k_e \cdot \omega$. The goodness-of-fit for a linear model is reflected in the small errors associated with the electric, torque, and speed constants. Here, it is once again noted that while the performance constants are reported in conventional units, standard S.I units must be used to correctly relate them to each other. Lastly, we note that the high time-resolution of the oscilloscope renders the horizontal error-bars negligible with respect to the relatively large uncertainty of the DMM readings.

4.3 No-Load and Stall Operation, Torque vs Speed

The no load speed (3350 ± 0.0002rpm) is in slightly larger than the maximum measured speed in the EMF measurement setup - this is consistent with the increased load on the shaft in the latter experiment. Once again, the error associated with the speed is very small due to the strong temporal resolution of the oscilloscope.

The estimated stall torque of 4.09 ± 0.1Ncm is very small, as expected of a brushed PMDC motor. This small torque is why DC motors are often used in conjunction with a gearbox. Note that the stall torque was evaluated using the torque constant and the stall current, not measured directly. However, the low uncertainty of the torque constant ($k_t = 6.04 \pm 0.03 \frac{\text{Ncm}}{\text{A}}$) gives this estimation a good error range.

With the two operating points at no-load and stall conditions known, the theoretical linear torque vs speed curve for a PMDC motor was formulated. Importantly, the maximum output power rating of 3.59 ± 0.1W is smaller than the 5W power rating. The maximum output power is not sustainable over longer periods of operation, as ohmic heating will resulting in the melting of the insulators and shorting in the coils. The more realistic estimate of peak efficiency (17) indicates that the motor is not very efficient even under optimal conditions. Several factors contribute to this inefficiency: running high currents through long coils introduces inevitable thermal losses. Additionally, brushed DC-motors suffer from added friction losses at the commutator.

4.4 Mechanical Time Constant

When deriving the mechanical time constant, we studied the motor's transient response to a step input. The tachometer readings eventually settled at a steady frequency of roughly 450 Hz, but upon further investigation, this is on the order of 27000 rpm. The motor shaft clearly did not turn at a speed on the same order of magnitude, so we suspect that there could be some multiplicative factor on the frequency of the tachometer that lead to this reading. However, since our approach was to determine when the system response reaches 63% of the steady response, this shouldn't have had an impact on our mechanical time constant.

5 Sample Calculations

5.1 Speed Calculation from Hall-Effect Sensor Response Period

Using a period of 0.0191s extracted from the edge detecting algorithm,

$$\begin{aligned} f &= \frac{1}{T} = \frac{1}{0.0191s} = 52.5 \text{ Hz} \\ v &= 60 \cdot 52.5 \text{ Hz} = 3140 \text{ rpm.} \end{aligned} \tag{17}$$

5.2 Uncertainty of Speed Approximation

Using the same data points as above,

$$\begin{aligned} \delta v &= 60 \cdot \frac{30}{1000000} \cdot (0.0934 \text{ s} + 0.0743 \text{ s}) \\ &= 3.02 \cdot 10^{-4} \frac{\text{m}}{\text{s}} \end{aligned} \tag{18}$$

5.3 Uncertainty of DMM Readings

We take an EMF voltage reading with the following numerical values:

$$\begin{aligned} \delta &= C_1 \cdot \text{Reading} + C_2 \cdot \text{Resolution} \\ V &= 19.96 \text{ V} \\ C_1 &= 0.005 \\ C_2 &= 2 \\ \text{Resolution} &= 0.01; \\ \delta &= 0.005 \cdot 19.96 \text{ V} + 2 \cdot 0.01 = 0.1198 \text{ V} \end{aligned} \tag{19}$$

5.4 Error Propagation for Inductance

First error in δ is propagated to the value of ζ.

$$\begin{aligned} \Delta\delta &= \left|\frac{\partial \delta}{\partial V_n}\right| \delta V_n + \left|\frac{\partial \delta}{\partial V_{n+1}}\right| \delta V_{n+1} \\ \delta &= 2 \pm 1 \\ \delta\zeta &= \left|\frac{\partial \zeta}{\partial \delta}\right| \Delta\delta \\ \zeta &= 0.3 \pm 0.1 \end{aligned}$$

Then, the error in the oscilloscope timing (from datasheet) is propagated to the calculation of damped frequency, ω_d.

$$\begin{aligned}
t_1 &= 5 \cdot 10^{-5} \text{ seconds} \\
t_2 &= 3.13 \cdot 10^{-4} \text{ seconds} \\
\delta t_1 &= \frac{30}{10^6} t_1 \\
\delta t_2 &= \frac{30}{10^6} t_2 \\
\delta T &= \delta t_1 + \delta t_2 \\
T &= 2.58 \cdot 10^{-4} \pm 10^{-8} \text{ seconds} \\
\delta \omega_d &= \frac{2\pi}{T^2} \delta T \\
\omega_d &= 2.44 \cdot 10^4 \pm 1 \frac{\text{rad}}{s}
\end{aligned}$$

Lastly, the error is propagated from both ζ and ω_d to ω_n and subsequently, L.

$$\begin{aligned}
\delta \omega_n &= \left| \frac{\partial \omega_n}{\partial \omega_d} \right| \delta \omega_d + \left| \frac{\partial \omega_n}{\partial \zeta} \right| \delta \zeta \\
&= \frac{1}{\sqrt{1-\zeta^2}} \delta \omega_d + \frac{\omega_d \zeta}{(1-\zeta^2)^{\frac{3}{2}}} \delta \zeta \\
\omega_n &= 25.3 \pm 1 \frac{\text{rad}}{s} \\
\delta L &= \left| \frac{\partial L}{\partial \omega_n} \right| \delta \omega_n + \left| \frac{\partial L}{\partial C} \right| \delta C \\
&= \frac{2}{\omega_n^3 C} \delta \omega_n + \left| \frac{1}{\omega_n^2 C^2} \right| \delta C \\
L &= 16 \pm 2\text{mH}
\end{aligned} \tag{20}$$

5.5 Maximum Power Calculation

$$\begin{aligned}
\omega_{noload} &= 350 \frac{\text{rad}}{\text{s}} \\
\tau_{stall} &= 4.09 \text{ Ncm} \\
P_{max} &= \frac{1}{2} \cdot \text{No-Load Speed} \cdot \frac{1}{2} \cdot \text{Stall Torque} \\
&= \frac{1}{2} \cdot \frac{\text{rad}}{\text{s}} \cdot \frac{1}{2} \cdot 4.09 \text{ Ncm} \\
&= 3.59 \text{ W}
\end{aligned} \tag{21}$$

5.6 Propagation of Uncertainty in Multiplication

We take the propagation of error in calculating the maximum power.

$$\begin{aligned} P_{max} &= 3.59 \text{ W} \\ \omega_{noload} &= 350 \pm 0.00002 \frac{\text{rad}}{\text{s}} \\ \tau_{stall} &= 4.09 \pm 0.1 \text{ Ncm} \\ \delta P_{max} &= P_{max} \cdot (\frac{(\delta\omega)}{\omega} + \frac{\delta\tau}{\tau}) \\ &= 3.59 \text{ W} \cdot (\frac{(0.0002)}{350} + \frac{0.001}{0.0409}) \\ &= 0.0878 \text{ W} \end{aligned} \tag{22}$$

5.7 Maximum Efficiency Calculation

$$\begin{aligned} I_o &= 64.8 \text{ mA} \\ I_s &= 678 \text{ mA} \\ \eta_{max} &= \left(1 - \sqrt{\frac{I_o}{I_s}}\right)^2 \\ &= \left(1 - \sqrt{\frac{64.8 \text{ mA}}{678 \text{ mA}}}\right)^2 \\ &= 0.477 \end{aligned} \tag{23}$$

6 Appendix

6.1 For Processing EMF vs Speed and Stall Point Data

```
%% Use EMF vs w data to predict motor response and account for uncertainty
clear;clc;clf;
%Define linear fit with sensitivity k
linFit = fittype(@(k,t)k.*t,...
    'independent',{'t'},'coefficients',{'k'});

%EMF voltages (via DMM) [V]
vEMF = [0.932,2.575,4.65,6.50,8.35,10.31,12.20,14.19,15.82,17.79,19.96];
vIn = 4:2:24; %Input voltage sweep [V]
vM = zeros(1,11); %Allocate space for motor speeds
vMError = zeros(1,11); %Allocate space for motor speed errors
vEMFError = zeros(1,11); %Allocate space for EMF errors

%Error propagation of DMM readings
a = 0.5./100;
b = 2;
res1_1 = 0.001;
res1_2 = 0.01;
%Based on DMM datasheet
vEMFError(:,1:2) = DMMError(a,b,res1_1,vEMF(:,1:2));
vEMFError(:,3:end) = DMMError(a,b,res1_2,vEMF(:,3:end));

for n = 1:11
    loadFile = sprintf('%i.csv',n); %Load files
    mat = readmatrix(loadFile); %Save file content
    A = mat(:,2); %Amplitude [V]
    t = mat(:,1); %Time [s]

    %Plot the measured voltages from the hall-effect sensor
    font = 20;
    figure();
    plot(t,A,'-k','Linewidth',0.5);
    set(gca,'fontsize',15);
```

```
xlabel('Time (s)','FontSize',font);
ylabel('Voltage (V)','FontSize',font);
title('Hall-Effect Sensor Output');
grid on;
saveFile = sprintf('%i.png',n);
saveas(gcf,saveFile);

if(A(1)<2) %Starting LOW
    %Logic for detecting HIGH square edge
    flow = 1; %Controls while loop
    i = 1; %Sweep through vector elements
    j = 1; %Starting cycle marker: LOW
    k = 0; %Trailing cycle marker: HIGH
    l = 0; %Cycle switch marker
    while flow == 1
        %Starting cycle: detect LOW -> HIGH
        if A(i)>2 && A(i+5)>2 && j == 1
            if l == 0
                tStart = t(i); %Start time
                j = 0;
                k = 1;
            end
            if l == 1
                tEnd = t(i-1); %End time
                flow = 0; %Terminating loop
            end
        end
        %Trailing cycle: detect HIGH -> LOW
        if A(i)<2 && A(i+5)<2 && k == 1
            j = 1;
            k = 0;
            l = 1; %If the low has been captured, terminate
        end
        i = i+1; %Increment through elements
    end
else %Starting HIGH
```

```
        %Logic for detecting HIGH square edge
        flow = 1; %Controls while loop
        i = 1; %Sweep through vector elements
        j = 1; %Starting cycle marker: LOW
        k = 0; %Trailing cycle marker: HIGH
        l = 0; %Cycle switch marker
        while flow == 1
            %Starting cycle: detect HIGH -> LOW
            if A(i)<2 && A(i+5)<2 && j == 1
                if l == 0
                    tStart = t(i); %Start time
                    j = 0;
                    k = 1;
                end
                if l == 1
                    tEnd = t(i-1); %End time
                    flow = 0; %Terminating loop
                end
            end
            %Trailing cycle: detect LOW -> HIGH
            if A(i)>2 && A(i+5)>2 && k == 1
                j = 1;
                k = 0;
                l = 1; %If the low has been captured, terminate
            end
            i = i+1; %Increment through elements
        end
    end

    T = (tEnd - tStart); %Period is double that of the half-period
    f = 1./T; %Frequency [rev/sec]
    vM(n) = f.*60; %Speed [rpm]
    %Horizontal (time) error propagation of oscilloscope
    vMError(n) = 60.*(30.*(1/1000000).*(abs(tEnd) + abs(tStart)));
end
```

```
%Estimate electric/torque constants
w = vM.*((2.*pi)./60); %[rad/s]
c = fit(w',vEMF',linFit); %Linear fit to data
cf = coeffvalues(c);
k_e = cf(1); %[V/(rad/s)]
k_t = k_e; %[Nm/A]
k_e = k_e .* 1000; %[mV/(rad/s)]
k_t = k_t.*100; %[Ncm/A]

%Error propagation from fit
err = confint(c); %Find interval of confidence
steError = (err(2,1) - err(1,1))/2; %Find uncertainty from interval
k_eError = steError(1);
k_tError = k_eError;
k_eError = 1000.*k_eError;
k_tError = 100.*k_tError;

%Plot the motor EMF as a function of speed using data points
font = 20;
figure();
plot(vM,vEMF,'-*k','Linewidth',1);
hold on;
e1 = errorbar(vM,vEMF,vEMFError);
e1.CapSize = 20;
e1.LineStyle = 'none';
hold on;
e2 = errorbar(vM,vEMF,vMError,'horizontal');
e2.CapSize = 20;
e2.LineStyle = 'none';
hold off;
set(gca,'fontsize',15);
xlabel('Motor Speed (rpm)','FontSize',font);
ylabel('EMF Voltage (V)','FontSize',font);
title('Back EMF as a Function of Motor Speed');
grid on;
saveas(gcf,'EMF_vs_SPEED.png');
```

```
%Stall current and torque calculations
c = 2.5./100; %Error coefficients
d = 5;
res2 = 0.001;
IStall = 678; %[mA]
IStallError = DMMError(c,d,res2,IStall); %Error propagation
tauStall = k_t.*(IStall./1000); %Stall torque [Ncm]
tauStallError = errorProd(k_e,k_eError,(IStall./1000),...
    (IStallError./1000),tauStall); %Error propagation

%Speed constant estimation
k_s = 1/cf(1); %[rad/s/V]
k_s = k_s.*(60./(2.*pi)); %[rpm/V]

%Error propagation from fit
k_sError = steError(1).*(60./(2.*pi)); %Find uncertainty from interval

%Error propagation from fit
err2 = confint(d); %Find interval of confidence
k_sError = (err2(2,1) - err2(1,1))/2; %Find uncertainty from interval
```

6.2 For Processing No Load Data

```
%% Use no-load data to predict motor response and account for uncertainty
clc;clear;
mat = readmatrix('hope_6_1.csv'); %Save file content
A = mat(:,2); %Amplitude [V]
t = mat(:,1); %Time [s]

%Plot the measured voltages from the hall-effect sensor
font = 20;
figure();
plot(t,A,'-k','Linewidth',0.5);
set(gca,'fontsize',15);
xlabel('Time (s)','FontSize',font);
ylabel('Voltage (V)','FontSize',font);
```

```
title('Hall-Effect Sensor Output');
grid on;
saveas(gcf,'No_Load_Speed.png');

if(A(1)<2) %Starting LOW
    %Logic for detecting HIGH square edge
    flow = 1; %Controls while loop
    i = 1; %Sweep through vector elements
    j = 1; %Starting cycle marker: LOW
    k = 0; %Trailing cycle marker: HIGH
    l = 0; %Cycle switch marker
    while flow == 1
        %Starting cycle: detect LOW -> HIGH
        if A(i)>2 && A(i+5)>2 && j == 1
            if l == 0
                tStart = t(i); %Start time
                j = 0;
                k = 1;
            end
            if l == 1
                tEnd = t(i-1); %End time
                flow = 0; %Terminating loop
            end
        end
        %Trailing cycle: detect HIGH -> LOW
        if A(i)<2 && A(i+5)<2 && k == 1
            j = 1;
            k = 0;
            l = 1; %If the low has been captured, terminate
        end
        i = i+1; %Increment through elements
    end
else %Starting HIGH
    %Logic for detecting HIGH square edge
    flow = 1; %Controls while loop
    i = 1; %Sweep through vector elements
```

```
    j = 1; %Starting cycle marker: LOW
    k = 0; %Trailing cycle marker: HIGH
    l = 0; %Cycle switch marker
    while flow == 1
        %Starting cycle: detect HIGH -> LOW
        if A(i)<2 && A(i+5)<2 && j == 1
            if l == 0
                tStart = t(i); %Start time
                j = 0;
                k = 1;
            end
            if l == 1
                tEnd = t(i-1); %End time
                flow = 0; %Terminating loop
            end
        end
        %Trailing cycle: detect LOW -> HIGH
        if A(i)>2 && A(i+5)>2 && k == 1
            disp(A(i))
            disp(i);
            j = 1;
            k = 0;
            l = 1; %If the low has been captured, terminate
        end
        i = i+1; %Increment through elements
    end
end

T = (tEnd - tStart); %Period is double that of the half-period
f = 1./T; %Frequency [rev/sec]
vNoLoad = f.*60; %No load Speed [rpm]
wNoLoad = vNoLoad.*((2.*pi)./60); %[rad/s]

%Horizontal (time) error propagation of oscilloscope
vNLError = 60.*(30.*(1/1000000).*(abs(tEnd) + abs(tStart)));
wNLError = vNLError.*((2.*pi)./60);
```

```
%No load current and error propagation
a = 1.5./100; %Error coefficients
b = 3;
res = 0.1;
INoLoad = 64.8; %[mA]
INLError = DMMError(a,b,res,INoLoad); %Error propagation

%Maximum efficiency
IStall = 678;
IStallError = 16.9550;
nuMax = (1 - sqrt(INoLoad./IStall)).^2;
%Error propagation: error propagation in squaring/square root cancel
nuMaxError = errorProd(INoLoad,INLError,IStall,IStallError,...
    (INoLoad./IStall));

%Torque vs Speed Curve
omega = [0,vNoLoad];
tau = [IStall,0];
%Plot the torque-velocity curve
font = 20;
figure();
plot(omega,tau,'-*k','Linewidth',1);
hold on;
e1 = errorbar(omega,tau,[IStallError,0]);
e1.CapSize = 20;
e1.LineStyle = 'none';
hold on;
e2 = errorbar(omega,tau,[0,vNLError],'horizontal');
e2.CapSize = 20;
e2.LineStyle = 'none';
hold off;
set(gca,'fontsize',15);
xlabel('Speed (rpm)','FontSize',font);
ylabel('Torque (Ncm)','FontSize',font);
title('Torque vs Speed Curve');
```

```
grid on;
saveas(gcf,'tau_vs_omega.png');

%Maximum Power
tauStall = 4.09404202511997;
tauStallError = 0.124566579703378;
pMax = (0.5.*wNoLoad).*(0.5.*tauStall./100);
pMaxError = errorProd(0.5.*wNoLoad,0.5.*wNLError,0.5.*tauStall./100,...
0.5.*tauStallError./100,pMax);
```